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STUDY ON DETERIORATING ITEMS AND LINEAR DEMAND INVENTORY MODEL WITH PROCUREMENT AND REPAIR

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Abstract: A deterministic inventory system for items with a constant deterioration rate is studied. Demand varies in time and it is assumed that it follows a power pattern. Shortages are allowed and backlogged. The ordering cost, the holding cost, the backlogging cost, the deteriorating cost, and the purchasing cost are considered in the inventory management. An approach is proposed to minimize the total cost per inventory cycle. This cost depends on two decision variables: the time at which the inventory level falls to zero and the length of the scheduling period. Numerical examples illustrate the theoretical results.

Keywords: Inventory Model, Deteriorating Items, Linear Demand, Repair

Introduction: In real world situations, it is observed that many products in life have a seasonal demand pattern. So the models with seasonal demand are prevalent because of its extensive application in the inventory management of the products with short life cycles. As a type of demand, seasonal demand is extremely common but is difficult to manage, efficiently. Bradely and Arntzen (1999) developed a model for seasonal demand environment, in which they simultaneously take capacity, inventory and scheduling decisions for maximizing the return on assets. Single opportunity for procurement and single option for switching the market have been considered by Petruzzi and Monahan (2003). They consider two non-overlapping markets with selling price lower in the secondary market. You (2005) considered pricing for an inventory model with deterministic price dependent seasonal demand declining with time. Hsu et al. (2007) presented an optimal ordering decision for deteriorating items with expiration date and uncertain lead time for seasonal products. In this they discussed the items whose demand decreases as they are nearer to the expiration date. Tayal et al. (2014) presented an inventory model of seasonal products for deteriorating items in which they apply a preservation technology cost to reduce the product's rate of deterioration. In this model the occurring shortages are partially backlogged. Saha S. et al. (2010) investigated an optimal pricing and production lot-sizing policy for seasonal products over a finite horizon.[1,2]
For today's high competitive market, immensity purchasing of inventory becomes convenient or obligatory and also results in significant cost reduction. Sometimes the overall circumstances are such that the supplier is influenced into buying more than he can sell in a season. Under such circumstances, he is compelled to rent another warehouse to stock the excess items or to transport them any other place where these are required. The seasons for the products may be different in different cities. For example crackers are used on different occasions at different regions in India. [3]

Woollens are start to use around September in the northern regions and then moves southwards till January. Hence, demand for different products experiences a boom at one place and slump at another place at the same time. Banerjee and Sharma (2010) investigated an inventory model for seasonal demand in which they introduced the option of an alternate market. According to the demand of the product distributor may sell the product in secondary market at higher or same price and thus the cost of warehouse and deterioration can be reduced. In general, almost all items deteriorate over time. [4]
There are many products in the real world that are subject to a significant rate of deterioration. Singh and Singh (2007) proposed an EOQ inventory model in which they considered the Weibull distribution deterioration, ramp type demand and partial backlogging. They optimized the order quantity, ordering cost, reorder point and lead time. Shukla et al. (2013) presented an EOQ model for deteriorating items with exponential demand rate and shortages. In this proposed model, shortages are allowed and partially backlogged. Tripathi and Mishra (2014) introduced an inventory model with inventory-dependent demand for deteriorating items in a single warehouse system. This paper derives a deterministic inventory model with single warehouse and shortages. [5] Pattnaik (2013) developed an inventory model for optimization in an instantaneous economic order quantity (EOQ) incorporated with promotional effort cost, variable ordering cost and units lost due to deterioration. Singh and Sharma (2014) presented an optimal trade-credit policy for perishable items deeming imperfect
production in which they considered the consumption rate as stock dependent. Tayal et al. (2014) introduced a multi item inventory model for deteriorating products and allowable shortages. In this model the effect of expiration date

is discussed. Singh et al. (2009) developed an inventory model for perishable items in which they considered the power demand pattern and partial backlogging of occurring shortages. In this model, they optimized the order quantity and replenishment cycle of the product. [6] Singh and Singh (2009) developed a production inventory model with variable demand rate for deteriorating items under permissible delay in payment. In this model a single item, single cycle economic production quantity model for perishable products is proposed where the demand is two component and stock dependent. Singhal and Singh (2013) worked on a volume flexible multi items inventory system with imprecise environment for deteriorating product. The decrease or loss of utility due to decay is usually a function of the on-hand inventory. It is reasonable to note that a product may be understood to have a lifetime which ends when utility reaches zero. Price is also an important factor that influences demand. Whitin (1955) first incorporated economic price theory in inventory literature. Saha and Basu (2010) developed an inventory model for deteriorating items with ramptype time and price dependent consumption rate for seasonal product over a finite planning horizon. Singh et al. (2011) worked on a soft computing based inventory model for deteriorating items in which they considered the consumption rate as price dependent. In this study they considered the two warehouse and solve this model under inflationary environment. [7]

Review of literature:
In the literature of inventory theory, it is often assumed that payment will be made to the vendor for the goods immediately after receiving the consignment. Such an assumption is not quite practical in the real world. Under most market behaviours, a vendor often provides a credit period to buyers in order to stimulate the demand, boost market share or decrease inventories of certain items. Recently, several researchers have devel- oped analytical inventory models with consideration of permissible delay in payments. [14] Goyal (1985) established a single-item inventory model under the condition of permissible delay in payments. Chung (1989) presented the discounted cash ¯ows (DCF) approach for the analysis of the optimal inventory policy in the presence of the trade credit. [15] Later, Shinn et al. (1996) extended Goyal's (1985) model and considered quantity discounts for freight cost. Chung (1997) presented a simple procedure to determine the optimal replenishment cycle to simplify the solution procedure described in Goyal (1985). Furthermore, it is also usually observed in the market that the sales of items like electronic components, fashion- able commodities, domestic goods, etc. increase rapidly after gaining consumer acceptance. Dave and Patel (1981) were the ®rst to address the problems with time- varying demand and a deterioration rate. They consid- ered a linear increasing demand rate over a ®nite horizon and a constant deterioration rate. Since this introduction, a number of studies, e.g. Murdeshwar (1988), Goswami and Chaudhuri (1991), Goyal et al. (1992), Hariga (1996), Chakrabarti and Chaudhuri (1997), Benkherouf (1995), and Chang and Dye (1999), devel- oped economic order quantity models that focused on deteriorating items with time-varying demand. However, in many inventory systems, the deterioration of goods is a realistic phenomenon. It is well known that certain products, e.g. medicine, volatile liquids, blood bank, food stuå and many others decrease under deterioration (vaporization, damage, spoilage, dryness, etc.) during their normal storage period. As a result, while determining the optimal inventory policy of that type of products, the loss due to deterioration cannot be ignored. In addition, it has been empirically observed that failure and life expectancy of many items can be expressed in items of Weibull distribution. [8,9] Hence, many authors have considered inventory models for this type of products. Covert and Philip (1973) obtained an economic order quantity model for a variable rate of deterioration by assuming a two-parameter Weibull distribution. Elsayed and Teresi (1983) extended their work by allowing for shortages and using time-varying demand rate. Concerning the Weibull distribution deterioration, related studies can be referred to Philip (1974), Wee (1997), Chen (1998) and Chakrabarty et al. (1998). [10] More recently, in order to agree with the practical inventory situation, Aggarwal and Jaggi (1995) consid- ered the inventory model with an exponential deterioration rate under the condition of permissible delay in payments. Hwang and Shinn (1996) addressed the joined price and lot size determination problem for an exponentially deteriorating product when the vendor permits delay in payments. Jamal et al. (1997) extended Aggarwal and Jaggi's (1995) model to allow for shortages. [11,12]

Assumptions regarding mthods

1. The retailers selling price per unit p and backorder price p_s are predetermined such that p_s =

$$\lambda p > c \text{ where } \lambda > 1$$

2. Seasonal pattern demand for the product follows a deterministic function of price and season such that

$$d_j(t,p)=\frac{aw(j)}{p_i^b}, \text{ j=1,2,……….N },$$
$$= 0 \qquad\qquad j > N$$

$$\text{where } w(j)=\frac{N-j+1}{N} \qquad a,b>0$$

This means that the customer's demand is smaller when it is nearer to the product expiration date.
3. Shortages are allowed and if possible, partially backlogged.
4. The fraction of customers backordered is assumed to be linearly decreasing with waiting time η and is assumed to be:-

$$\theta(\eta)=1-\frac{\eta}{T} \qquad\qquad 0\le\eta\le T$$

5. The warehouse has unlimited capacity.
6. Lead time is considered.
7. Deterioration rate is taken as a Weibull function of time.
8. At the end of each season in the primary market, the distributor has an option of transferring the remaining inventory to the alternate market and/or change the price. Once the inventory is in the alternate market, it cannot be brought back to the primary market.
9. Supplier's delivery does not go beyond the second season
10.The time lag and deterioration during transportation is negligible.

Mathematical model: Suppose a manufacturer produces a certain product and sells it in a market. All items are produced and sold in each cycle. The following assumptions are used to formulate the problem. [13]

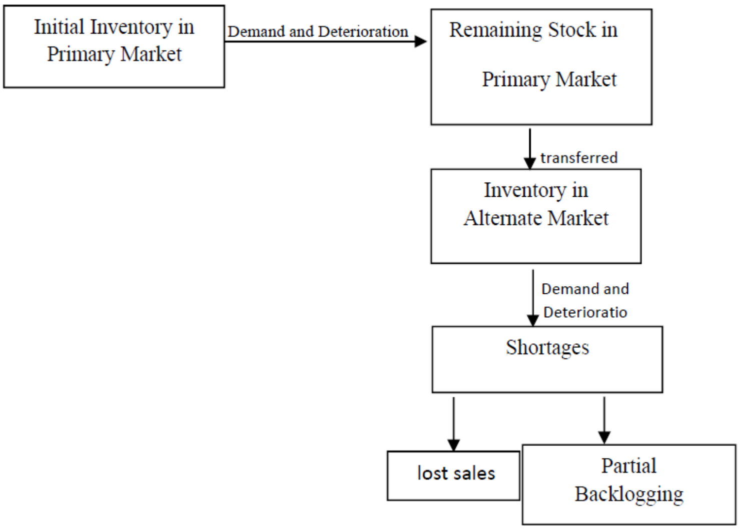

Fig. 1. Flow chart of inventory.

The Basic Model without Disruptions

At first, the manufacturer makes decisions about the optimal production time *T*p under the normal production rate. The inventory model for deteriorating items with normal production rate can be depicted as in Figure 2.

The instantaneous inventory level at any time $t \in [0,H]$ is governed by the following differential equations:

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = p - d, \quad 0 \le t \le T_p,$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -d, \quad T_p \le t \le H.$$

Using the boundary condition $I_1(0) = 0$ and $I_2(H) = 0,$ solutions of above differential equations are

$$I_1(t) = \frac{p-d}{\theta}\left(1 - e^{-\theta t}\right), \quad 0 \le t \le T_p,$$

$$I_2(t) = \frac{d}{\theta}\left[e^{\theta(H-t)} - 1\right], \quad T_p \le t \le H.$$

The condition $I_1(T_p) = I_2(T_p)$ yields

$$\frac{p-d}{\theta}\left(1 - e^{-\theta T_p}\right) = \frac{d}{\theta}\left[e^{\theta(H-T_p)} - 1\right]$$

From (2.3), the production time T_p satisfies the following equation:

$$T_p = \frac{1}{\theta}\ln\frac{p - d + de^{\theta H}}{p}.$$

In order to facilitate analysis, we do an asymptotic analysis for $I_i(t)$. Expanding the exponential functions and neglecting second and higher power of θ for small value of θ.

$$I_1(t) \approx (p-d)\left(t - \frac{1}{2}\theta t^2\right), \quad 0 \le t \le T_p,$$

$$I_2(t) \approx d\left[(H-t) + \frac{1}{2}\theta(H-t)^2\right], \quad T_p \le t \le H,$$

and T_p approximately satisfies the equation

$$(p-d)\left(T_p-\frac{1}{2}\theta T_p^2\right)=d\left[(H-T_p)+\frac{1}{2}\theta(H-T_p)^2\right]$$

From Misra [31], we have

$$T_p \approx \frac{d}{p-d}(H-T_p)\left[1+\frac{1}{2}\theta(H-T_p)\right]$$

we can get the following corollary.

Assuming that $\theta \ll 1$, then T_p is increasing in θ.

The manufacturer has to produce more products when deterioration rate increases. Hence, decreasing deterioration rate is an effective way to reduce the product cost of manufacture.

Conclusions

In this paper, we propose a production-inventory model for a deteriorating item with production disruptions. Here, we analyze this inventory system under different situations. We have showed that our method helps the manufacturer reduce the loss caused by production disruptions. In this study, the proposed model considers the deterioration rate as constant. In real life, we may consider the deterioration rate as a function of time, stock, and so on. This will be done in our future research.

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