STUDY ON THE REVIEW OF LITERATURE OF FUZZY GRAPH

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Accepted: 06.03.2023 **Published**: 10.04.2023

Abstract: A graph which admits graceful labelling is called graceful graph. Various kinds of graphs are shown to be graceful. A magic square is an arrangement of numbers into a square such that the sum of each row, column and diagonal are equal. The term antimagic then comes from being the opposite of magic or arranging numbers in a way such that no two sums are equal [77]. The interest in graph labelling can trace its roots back to a paper by A. Rosa in 1966. Nora Hartsfield and Gerhard Ringel introduced the concept of antimagic labelling in 1990 which is an assignment of distinct values to different objects in a graph such a way that when taking certain sums of the labels the sums will all be different. A weak antimagic labelling is similar to an antimagic labelling except one does not reqire distinct labels. Bipartite graphs have a weak antimagic labelling while limiting the labels of the edgesto a value less than or equal to the number of edges in the graph [77]. Bodendiek and Walter defined in 1994,the concept of an antimagic labelling as an edge labelling in which the vertex values form an arithmetic progression starting from a and have common difference d. Martin Baca, Francois Bertault and MacDougall (2003) introduce the notions of the vertex antimagic labelling in 2003 and Nissankara (2014) derived the algorithm for vertex antimagic labelling.

Keywords: Literature, Fuzzy graph, fuzzy bridge and fuzzy cut node

Introduction: Fuzzy graphs have many more applications in modelling real time systems where the level of information inherent in the system varies with different levels of precision [34]. Neural networks are simplified models of the biological nervous system and therefore have drawn their motivation from the kind of computing performed by a human brain. Neural networks exhibit characteristic such as mapping capabilities or pattern association, generalization, robustness, fault tolerance, and parallel and high speed information processing. Fuzzy neural networks and neural fuzzy systems are powerful techniques for various computational and control applications. The area is still under a great influx from both theoretical and applied research. There is no systematic or unified approach for incorporating the concepts of fuzziness and neural processing. Fuzzy sets can be used to describe various aspects of neural computing. That is, fuzziness may be introduced at the input output signals, synaptic weights, and aggregation operation and activation function of individual neurons to make it fuzzy neuron. Different aggregation operations and activation functions result in fuzzy neurons with different properties. Thus there are many possibilities for fuzzification of an artificial neuron. A fuzzy graph is the generalization of the crisp graph.Therefore it is natural that many properties are similar to crisp graph and also it deviates at many places . A fuzzy graph has ability to solve uncertain problems in a range of fields that'swhy fuzzy graph theory has been growing rapidly and consider it in numerous applications ofvarious fields like mathematical sciences and technology [85]. The first definition of a fuzzy graph was introduced by Kaufmann in 1973 and the structure of fuzzy graphs developed by Azriel Rosenfeld in 1975. Rosenfeld introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. There are many problems, which can be solved with the help of the fuzzy graphs.

Review of literature

In 1975, Rosenfeld considered fuzzy relations on fuzzy sets and developed the structure of fuzzy graphs, obtaining analogues of several graph theoretical concepts. He also introduced and examined such concepts as clusters, bridges, cut vertices, forests and trees [78]. Bhattacharya discussed some properties of fuzzy graphs and introduced the notion of eccentricity and centre in fuzzy graphs. He gave examples indicating that results from crisp graph theory do not always have analogue for fuzzy graphs. Bhutani introduced the concept of complete fuzzy graphs in 2003 and concluded that a complete fuzzy graph has no cut nodes. Applying fuzzy methods into the workings of neural networks constitutes a major thrust of neuro-fuzzy computing. A fuzzy neuron has the same basic structure as the artificial neuron except that its components and parameters are described through the mathematics of fuzzy logic. There are many possibilities for fuzzification of an artificial neuron so we may find a variety of fuzzy neurons in the literature. A fuzzy neuron consists of external inputs, synapses/synaptic weights, dendrites, soma, and an axon through which the neural output is transmitted to other neurons. The external inputs $x_1, x_2, .., x_n$ enter the j^{th}neuron and gets modified by the synaptic weights $w_{j1}, w_{j2}, ..., w_{jn}$. Each synaptic output forms an input to the processing element (soma), called the dendrite input. The external input x_1, x_2, ., $x_n$are of fuzzy signals bounded by membership values over the interval [0, 1]. The synaptic weights $w_{ji}$are also defined over the interval [0, 1]. An artificial neural network can be represented by a directed graph also. The vertices of the graph may represent neurons (input/ output) and the edges are labelled by the weights attached to the synaptic links. Now let S is a non empty set.In a fuzzy neural network we take each neuron as a node with unit weight and the relationships (connections) between the neurons (only forward direction) as arcs with weight (specifying the strength of connectivity) in the interval [0, 1].

As per Strahler's stream ordering principle, in Geography, numbers (orders) are assigned on the basis of a number of things like cost, distance, travel time, traffic loading etc. to the edges between vertices (places).For applying this in network, pen thickness is used to denote stream order. This sometimes affects the connectivity of the network by detaching some vertices which otherwise cannot remain isolated. For example, lakes which are origins of certain rivers get detached in this way. To avoid this, we can treat them as fuzzy edges of the fuzzy graph with membership grades representing stream order. Further, to denote different characteristics and to study them simultaneously, we can use different relations and

for each stream, we may use only that Ri-edge which is prominent. Thus we have a fuzzy graph structure to represent a generalised map. In a graph representing a map of an urban area, vertices denote places and edges denote connectivity. Orders are given to vertices and edges. Here also fuzzy edges with membership representing order will make the study easier and natural. Different types of connectivity may be represented by different relations. Thus here also we are presented with a fuzzy graph structure. Connectivity of a town to other parts of the world may be through roads, through roads and rail, through roads and river etc. These may be denoted by different relations of a graph structure with Ri-edges not belonging to more than one relation. The importance of each connection can be given by membership of the fuzzy edges of the corresponding fuzzy graph structure.

Fuzzy graph has ability to solve uncertain problems in a range of fields that's why fuzzy graph theory has been growing rapidly and consider it in numerous applications of various fields. Exponential growth in the theory has been showing its utilization within mathematical sciences as well as it brings into play important role in technology. A fuzzy graph contains many properties similar to crisp graph due to generalization of crisp graphs but it diverges at many places.

Different real-world problems may be illustrated by union of different dots all the way joined through lines. A mathematical notion of such illustrations turned into the concept of a graph theory. In such illustration, the dots may stand for cities, computers, pieces of lands or group of persons and lines may be a symbol of roads linking between different cities, cable connection for network between different computers, bridges connecting pieces of land or correlation between people. In graph theory nodes are known as vertices while the lines can be named arcs or edges [5].

Different cities, towns and villages will have different mean degrees and completeness. Graph theory helps to prevent road network of a town below a level at which town becomes disconnected. This would de-emphasize the difference in urban street complexity. According to Mackaness and Beard, connectivity has more importance than size. As rightly pointed out by them, fuzzy sets and fuzzy graphs are better able to represent these things. Fuzzy set theory can be used to avoid discrete constraints of structures. Further, fuzzy graph structure helps to represent different aspects simultaneously.

Fuzzy labelling has been studied by A.Nagoor Gani and D.Rajalaxmi (a) subahashini who discussed fuzzy labelling in magic graph in 2014. A research on fuzzy labelling has been witnessing an exponential growth, both within mathematics and in its applications in science and technology- especially in the fields of information theory, neural networks, expert systems, cluster analysis, medical diagnosis, control theory, etc [114]. In a social network, different relationships between pairs can easily be represented by a graph structure. Magnitude of a relationship, presently marked by thickness of arrows in a path diagram (with variables or groups as circles and relationships as arrows) can be replaced by membership values. This will form a fuzzy graph with individuals as vertices and relationship as fuzzy edges. We can represent different relationships by different fuzzy relations thereby representing the whole problem as a fuzzy graph structure.

Factor plots in network analysis, plots represent variables or groups as points on one or more two dimensional scatter plots, where dimensions are factors. The relationships in different dimensions may be represented by different relations of a graph structure. As in the previous case, magnitude of a relationship can be replaced by membership values and the problem can be converted to a problem of fuzzy graph structures[70].

In graph theory, graph labelling plays a vital role by using graph labelling; we can easily understand the graph. Structure mining or structured data mining is the process of finding and extracting useful information from semi structured sets of data. Graph mining is a special case of structured data mining. In data mining graphs can be more widely used because the outputs of data mining can be represented in graphs[70].

Networks representation plays an important role in many domains of computer science, ranging from data structures and graph algorithms, to parallel and distributed computing, and communication networks. Now a days database management is most efficiently used in many applications. In this data exists in the tables can be taken as nodes and then draw connections between the nodes what type of relationship exists can be taken as labelling in nodes[70].

Encryption has fascinated extra responsiveness owing to the swift development in multimedia and network technologies where the data can be shielded from unauthorized access. Image scrambling is a scheme that provides protection for digital images. The results point out that the new image scrambling method established on graceful labelling of tree can deliver a high level secure owing to the strong anomaly of sorting transformation[70]. The concept of public key cryptography proved extremely useful to solve problems coming up with the possibilities of the Internet and it is realized not only secure communication but for example digital signatures or digital authentication protocols as well.Security labels convey

information used by protocol entities to determine how to handle data communicated between open systems. Information on a security label has been used to control access, specified protective measures, and determined additional handling restrictions required by a communications security policy. In communication networks also it is applied because the route establishment, allocation of channels, security also provided in the networks.[70]

Hartsfield and Ringel [47] introduced antimagic graphs in 1990. Hefetz [50] calls a graph with q edges k-antimagic if its edges can be labelled with 1,2,...,q+k such that the sumsof the labels of the edges incident to each vertex are distinct. In particular, antimagic is the same as 0-antimagic. More generally, given a weight function w from the vertices to the natural numbers Hefetz [50] calls a graph with q edges (w,k) antimagic if its edges can be labelled with 1,2,...,q+k such that the sums of the labels of the edges incident to each vertex and the weight assigned to each by ware distinct. In particular, antimagic is the same as (w,0)-antimagic where w is the zero function. The concept of an (a,d) antimagic labelling was introduced by Bodendiek and Walther[28] in 1993.

Bertault , Miller , Pe-Roses , Feria-Puron and Vaezpour [22] approached labelling problems as combinatorial optimization problems. They developed a general algorithm to determine whether a graph has a magic labelling , antimagic labelling or an (a,d)-antimagic labelling.

The concept of a graceful labelling has been introduced by Rosa in 1967 and named itas a β - valuation of graph while Golomb independently introduced such labelling and called it as graceful labelling. Acharya has constructed certain infinite families of graceful graphs from a given graceful graph in 1982 while Rosa and Golomb have discussed gracefulness of complete bipartite graphs and eulerian graphs in 2001.Acharya proved that every graph can be embedded as an induced sub graph of a graceful graph and a connected graph can be embedded as an induced sub graph of a graceful connected graph [1].

The odd graceful labeling is one of the most widely used labeling methods of graphs . While the labeling of graphs is perceived to be a primary theoretical subject in the field of graph theory and it serves as models in a wide range of applications [71].

Gracefulness of union of two path graphs with grid graphs and complete bipartite graphs are discussed in by Vaidya in 2008. Also many variations of graceful labelling have been introduced in recent years by researchers. Graceful labelling for complete bipartite graphs has been discussed by V.J.Kaneria, H.M.Makadia, M.M.Jariya and Meera Meghapara in 2008.

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