OPTIMIZATION AND EQUITABLE DOMINATION IN BIPARTITE AND MULTI-LAYERED NETWORKS: ALGORITHMS, COMPLEXITY, AND APPLICATIONS

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Abstract
Dominating-set theory provides a rigorous way to select a small number of vertices that can cover, influence, monitor or control an entire network. In conventional graph theory, a dominating set is evaluated primarily by coverage; however, many practical networks also require fairness between selected and covered vertices. This paper examines optimization and equitable domination in bipartite and multi-layered networks, with emphasis on algorithms, computational complexity and applications. The study adopts a descriptive-analytical and conceptual mathematical research design. It reviews domination variants, degree equitable domination, k-tuple domination, bipartite coverage models and multilayer network representations, and it synthesises them into an optimization framework. The paper argues that equitable domination is especially useful when network service should not be concentrated only on high-degree hubs but should also preserve local degree compatibility, balanced access and layer-aware coverage. In bipartite networks, the problem models allocation between two distinct classes, such as facilities and users, sensors and targets, or workers and tasks. In multilayer networks, the same logic extends to interconnected infrastructures, multiplex social relations, cyber-physical systems and transport-service layers. The analysis shows that exact optimization is generally computationally difficult because domination and several domination variants are NP-hard, while useful tractability can be obtained through greedy heuristics, integer programming, approximation strategies and parameterized algorithms for structured graph classes. The study concludes that equitable domination is not merely a theoretical refinement; it is a bridge between graph optimization, fairness-aware resource allocation and resilient network design.
Keywords: equitable domination; bipartite networks; multilayer networks; graph optimization; domination number; algorithms; complexity; resource allocation; network science

1. Introduction

Networked systems are now a dominant mathematical language for describing interactions in commerce, communication, transportation, biology, public administration and digital infrastructure. A network may represent people connected by information flows, sensors connected to monitored targets, health facilities connected to communities, or servers connected across layers of physical and virtual infrastructure. In such settings, one recurrent question is how to select a limited set of vertices that can efficiently cover the remaining system. The classical domination problem answers this question by identifying a subset of vertices such that every vertex outside the subset is adjacent to at least one selected vertex. Although standard domination is powerful, it does not always capture the equity requirements of contemporary network design. A selected high-degree hub can dominate many peripheral vertices, but this may produce a structurally unbalanced solution if the selected and dominated vertices have very different degrees, capacities or roles. Degree equitable domination responds to this concern by requiring that a vertex outside the dominating set be covered by a selected adjacent vertex whose degree differs by at most one. This additional condition transforms domination from pure coverage into compatible coverage (Swaminathan & Dharmalingam, 2011).

The relevance of equitable domination becomes stronger in bipartite and multilayer networks. Bipartite networks separate vertices into two classes and allow edges only across classes. They naturally arise in assignment, recommendation systems, labour markets, user-facility allocation and incidence structures. Multi-layered networks, in contrast, represent systems with different kinds of interactions, time periods or subsystems. Kivelä et al. (2014) noted that many real systems cannot be understood adequately through a single-layer graph because multiple relationship types can shape the same set of entities. When domination is formulated in such environments, the problem becomes not only to cover vertices but also to decide which layer, role and degree relationship should govern coverage. This paper develops a conceptual and algorithmic framework for studying optimization and equitable domination in bipartite and multi-layered networks. It does not claim to introduce a new theorem; rather, it integrates existing ideas from domination theory, equitable domination, k-tuple domination, parameterized algorithms and multilayer network science to show how equitable domination can be used as a modelling tool in complex systems. The paper is therefore positioned as a synthesis paper for mathematical and computational research.

1.1 Statement of the Problem

Dominating-set optimization in bipartite and multilayer networks raises three connected problems. First, the solution must cover the whole network with as few selected vertices as possible. Second, the coverage should be equitable in the sense that selected and covered vertices should not differ drastically in local degree or structural capacity. Third, the algorithm should be computationally feasible for networks that may contain thousands or millions of vertices. The central problem examined here is therefore: how can equitable domination be formulated and optimized in bipartite and multi-layered networks while preserving algorithmic tractability and practical interpretability?

1.2 Objectives of the Study

- To explain the mathematical foundations of domination, equitable domination and domination number in graphs.
- To examine how equitable domination modifies ordinary domination through degree compatibility.
- To analyse the structure of bipartite networks and identify why domination problems are useful in two-mode systems.
- To extend the domination perspective to multilayer and multiplex network settings.
- To compare exact, heuristic, approximation and parameterized approaches for domination optimization.

2. Conceptual and Mathematical Background

The formal starting point is a finite, simple and undirected graph G = (V, E), where V is a set of vertices and E is a set of edges. The open neighbourhood of a vertex v is denoted by N(v), and its degree is deg(v) = |N(v)|. A set D ⊆ V is a dominating set if every vertex outside D has at least one neighbour in D. The domination number γ(G) is the minimum cardinality of such a set. This concept is useful because it turns a network coverage task into a discrete optimization problem.

$$\mathbf{D \subseteq V(G) \text{ is dominating if } \forall v \in V(G) - D,\ N(v) \cap D \neq \emptyset.}$$

$$\boldsymbol{\gamma(G) = \min\{|D| : D \text{ is a dominating set of } G\}.}$$

Equitable domination adds a degree-based compatibility condition. A vertex u is said to dominate a vertex v equitably if u and v are adjacent and the difference between their degrees is at most one. A degree equitable dominating set is a set D such that each vertex outside D is equitably dominated by some vertex in D. This idea, introduced and developed in the degree-equitable domination literature, is important because it prevents coverage from being defined only by adjacency and introduces a fairness-sensitive local structural criterion (Swaminathan & Dharmalingam, 2011).

$$\mathbf{D \text{ is equitable dominating if } \forall v \in V - D,\ \exists u \in D \text{ such that } uv \in E \text{ and } |\deg(u) - \deg(v)| \leq 1.}$$

$$\boldsymbol{\gamma_e(G) = \min\{|D| : D \text{ is an equitable dominating set of } G\}.}$$

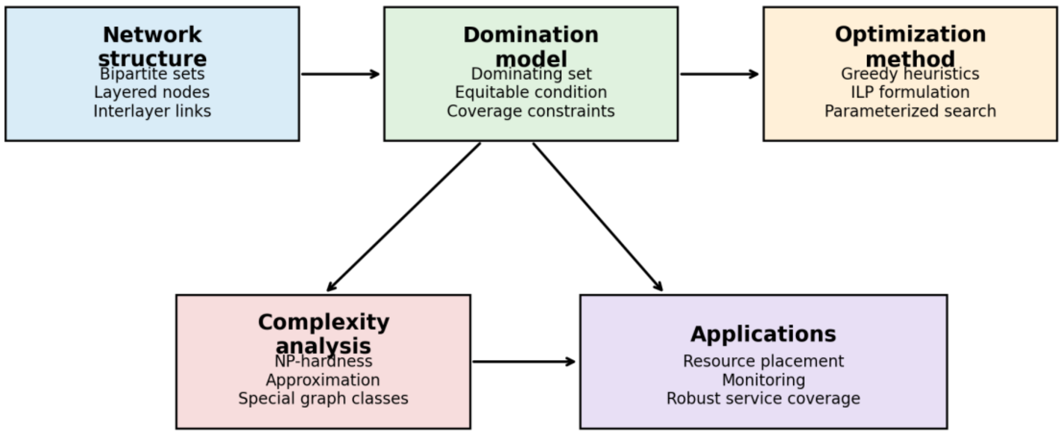
In bipartite graphs, the vertex set is divided into two disjoint parts U and W, and edges occur only between the two parts. This structure is valuable for modelling relationships where two different entity types interact. In a user-facility network, U may represent facilities and W may represent communities. In a recommendation network, U may represent users and W may represent items. Dominating sets in bipartite graphs can therefore represent minimum service points, selected representatives or influential objects.

$$\mathbf{G = (U \cup W, E),\ U \cap W = \emptyset,\ E \subseteq U \times W.}$$

A multilayer network introduces an additional layer dimension. Instead of representing only one type of edge, the network contains several layers, such as physical contacts, digital interactions, transport connections, or temporal states. De Domenico et al. (2013) provided a mathematical formulation of multilayer networks through tensorial representations, while Kivelä et al. (2014) offer a broad terminology and methodological framework for multilayer systems. In domination terms, this means that a vertex may be covered within one layer, across layers, or under both intra-layer and inter-layer conditions.

$$\mathbf{M = (V\_M, E\_M, V, L), \text{ where } V\_M \subseteq V \times L \text{ and } E\_M \subseteq V\_M \times V\_M.}$$

Figure 1. Conceptual framework linking network structure, equitable domination, optimization and applications.



3. Review of Literature

The literature on domination in graphs is extensive, but recent work has increasingly moved toward variants that capture stronger coverage, independence, connectivity, security and fairness. Liao and Chang (2002, 2003) studied algorithmic aspects of k-tuple domination, where each vertex requires multiple domination relationships. This literature is useful for the present study because equitable domination similarly modifies standard domination by adding a constraint beyond simple adjacency.

Klasing and Laforest (2004) established hardness and approximation results for k-tuple domination, showing that even apparently modest extensions of domination can generate difficult algorithmic problems. Their work is relevant because equitable domination also inherits the combinatorial complexity of domination while adding degree-compatibility restrictions. Cooper et al. (2004) studied dominating sets in web graphs, demonstrating that domination can be meaningful in large-scale networks where selected vertices may serve as navigational or control representatives. Degree equitable domination was developed by Swaminathan and Dharmalingam (2011), who defined and studied domination under degree equitability. Their work provides the direct foundation for this paper. Sivakumar et al. (2012) extended the idea toward connected equitable domination, highlighting the importance of maintaining both coverage and structural connectedness. Vaidya and Pandit (2013, 2016) developed related global and wheel-related equitable domination results, further showing that equitable conditions can be studied across special graph families.

Network science literature is equally important. De Domenico et al. (2013) showed that multilayer networks require mathematical frameworks beyond ordinary adjacency matrices. Kivelä et al. (2014) clarified the vocabulary of multilayer, multiplex and interconnected networks and showed why a single-layer representation can conceal relevant structure. These contributions allow domination theory to be interpreted in modern systems where interactions differ by type, medium or time.

Algorithmic complexity literature provides the computational background. Cygan et al. (2015) present parameterized methods for hard graph problems, including problems related to domination. Henning et al. (2019) investigated semipaired domination and established both hardness and tractable cases. Kumar and Reddy (2020) studied algorithmic aspects of 2-secure domination, while Lokshtanov et al. (2021) demonstrated fixed-parameter tractability of dominating set in weakly closed graphs. Together, these studies suggest that domination variants are generally difficult but may become solvable under restricted structures or parameters.

Recent work continues to expand equitable domination. Menon (2025) studies equitable domination numbers for graph operators, suggesting that the field remains active and that equitable variants can be extended to transformed graph structures. For bipartite and multilayer networks, this trend implies a need to connect equitable domination with realistic layered and two-mode network models rather than treating it only as an abstract invariant.

Table 1. Thematic synthesis of selected literature

Theme	Main contribution	Relevance to present study	Representative sources
k-tuple domination	Multiple coverage requirements in graphs	Shows how domination changes under stronger service conditions	Liao & Chang (2002, 2003); Klasing & Laforest (2004)
Equitable domination	Coverage under degree compatibility	Provides the fairness-sensitive foundation	Swaminathan & Dharmalingam (2011); Sivakumar et al. (2012)
Multilayer networks	Layer-based representation of complex systems	Supports extension beyond single-layer graphs	De Domenico et al. (2013); Kivelä et al. (2014)
Algorithmic complexity	Hardness, approximation and parameterized solutions	Guides method selection for large networks	Cygan et al. (2015); Lokshtanov et al. (2021)
Applications	Service, monitoring and influence problems	Connects theory to operational network design	Cooper et al. (2004); Henning et al. (2019)

4. Research Methodology

This paper uses a descriptive-analytical mathematical research design. It is descriptive because it presents the definitions, variants and structures relevant to domination and equitable domination. It is analytical because it compares algorithmic methods, explains complexity implications and develops a framework for applying equitable domination to bipartite and multilayered networks. The paper is not based on primary survey data; rather, it is based on conceptual synthesis of author-based mathematical and computational literature published between 2002 and 2025. The unit of analysis is the network represented as a graph or multilayer graph. The main variables are coverage, degree compatibility, layer coupling, algorithmic cost, and application suitability. The research proceeds in four steps: defining the mathematical objects; translating equitable domination into optimization constraints; comparing algorithmic approaches; and interpreting applications in practical network systems.

Table 2. Methodological framework

Component	Description	Purpose	Output
Research type	Conceptual and analytical	To synthesise mathematical literature	Integrated framework
Core object	Bipartite and multilayer networks	To represent two-mode and layered systems	Formal notation and diagrams
Optimization lens	Minimum equitable dominating sets	To model fair coverage	ILP-style constraints
Algorithmic lens	Exact, greedy, approximation and FPT methods	To compare tractability	Method selection table
Limitations	No empirical field data or new theorem proof	To define scope of claims	Conceptual recommendations

5. Optimization Formulation

A useful way to analyse equitable domination is to convert it into a binary optimization model. Let each vertex i have a decision variable x_i, where x_i = 1 if the vertex is selected in the dominating set and x_i = 0 otherwise. Ordinary domination requires every unselected vertex to be adjacent to at least one selected vertex. The equitable version restricts the eligible neighbours to those whose degrees differ by at most one. The objective is to minimize the total number of selected vertices.

$$\textbf{minimize} \;\; Z = \Sigma_i x_i$$

$$x_i + \Sigma_j \in N(i), |deg(j) - deg(i)| \leq 1 \; x_j \geq 1, \;\; \text{for every } i \in V.$$

The constraint states that vertex i is covered if it is selected itself or if at least one degree-compatible neighbour is selected. This formulation clarifies why equitable domination is usually more restrictive than ordinary domination. If a vertex has many neighbours but none of them have comparable degree, ordinary domination may cover it easily but equitable domination may require the vertex itself to be selected.

For bipartite networks, the optimization can be modified to select vertices from one partition, both partitions, or a restricted service side. In facility-location terms, one may select only facilities from U to cover clients in W. In more general bipartite domination, vertices from both sides may be selected. The equitable condition can still be enforced by comparing degrees across partitions.

$$\textbf{minimize } Z\_B = \Sigma_u \in U\ x_u + \Sigma^w \in W\ x^w$$

$$x_v + \Sigma_u \in N(v),\ |deg(u) - deg(v)| \leq 1\ x_u \geq 1,\ \text{ for } v \in U \cup W.$$

For multilayer networks, each vertex-layer pair is treated as a node-state. A decision variable $x_{i\alpha}$ indicates whether entity i in layer α is selected. Coverage can occur inside the same layer or across layers, depending on the application. For example, a transport station may cover nearby stations within the road layer and also connect through a metro layer. The formulation therefore includes intra-layer and inter-layer neighbours.

$$\textbf{minimize } Z\_M = \Sigma_{(i,\alpha)\in V\_M}\ x_{i\alpha}$$

$$x_{i\alpha} + \Sigma_{(j,\beta)\in N\_M(i,\alpha),\ |deg(j,\beta) - deg(i,\alpha)| \leq 1}\ x_{j\beta} \geq 1.$$

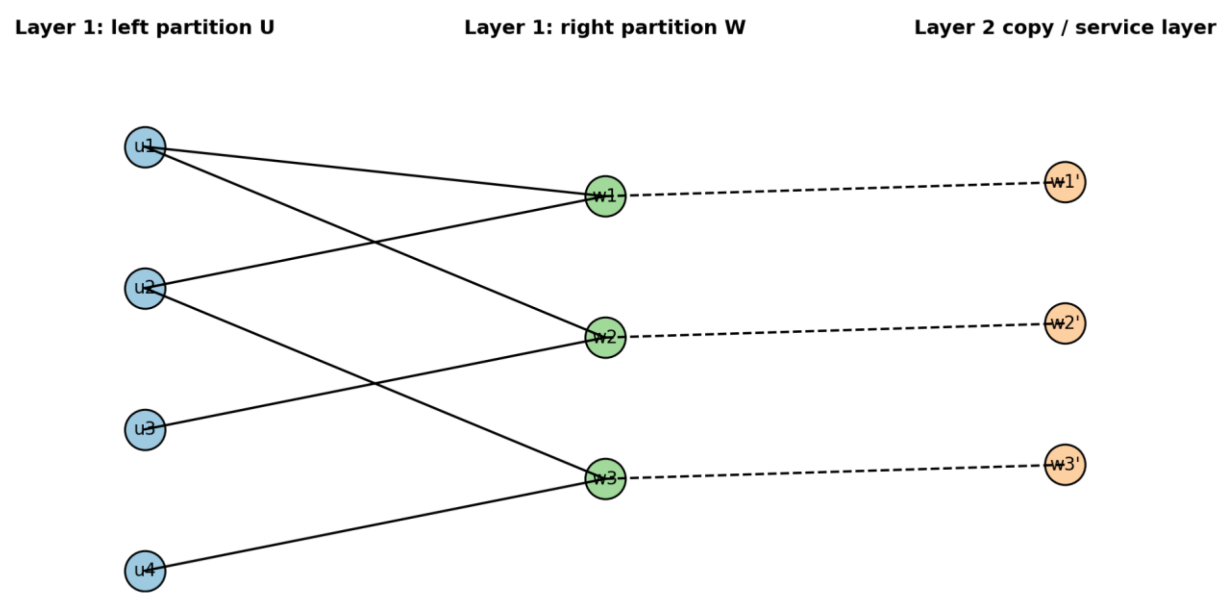
These formulas do not remove computational difficulty, but they provide a clear modelling structure. They also allow researchers to use integer programming solvers for small and medium networks, heuristic algorithms for large systems, and parameterized algorithms where the target domination number or treewidth is small.

6. Bipartite and Multi-Layered Network Interpretation

Bipartite networks are structurally important because they represent interactions between unlike categories. A two-mode network of hospitals and villages, buyers and sellers, students and courses, or sensors and targets cannot be represented accurately by a single homogeneous vertex type without losing role information. Domination in bipartite networks can therefore be interpreted as a coverage problem across roles. Equitable domination further asks whether a selected service vertex is structurally compatible with the vertex it covers. In a practical setting, this may correspond to matching capacity with demand or matching local influence with local exposure.

Multi-layered networks introduce a different kind of complexity. The same entity may exist in more than one layer, and each layer may contain a different relation. A person may be connected in a workplace layer, a family layer and a digital communication layer. A city may contain road, rail, power and digital layers. If domination is computed only on an aggregated graph, a vertex may appear powerful because it has many total links, but that power may not exist in the layer where coverage is needed. Layer-aware equitable domination therefore helps avoid misleading aggregation.

Figure 2. Schematic of bipartite coverage with interlayer coupling.



Solid edges show bipartite coverage; dashed edges show interlayer coupling.

The schematic illustrates the core modelling logic. Solid edges represent bipartite coverage within a base layer. Dashed edges represent coupling to a second layer. An ordinary solution might select a highly connected vertex even if it belongs to the wrong layer or degree class. A layer-aware equitable solution requires the selected vertex to be both adjacent and compatible. This is useful in public-service networks, cyber-physical systems and resilient infrastructure, where cross-layer interactions can create hidden dependencies.

7. Algorithms for Equitable Domination

The most direct method for computing equitable domination is exact search. The model is formulated as a 0–1 integer programme, and a solver searches for the minimum feasible set. Exact methods are valuable because they produce optimal solutions and can be used to validate heuristics. Their weakness is scalability. Because domination-related problems are generally computationally difficult, exact models can become slow as the number of vertices increases. Greedy algorithms offer a faster alternative. A greedy method repeatedly selects the vertex that covers the largest number of currently uncovered, degree-compatible vertices. In a multilayer setting, the greedy score can be modified by adding weights for interlayer importance, redundancy or application priority. Although greedy methods do not guarantee optimality in general, they are often interpretable and practical for large networks.

$$\text{score}(v) = |\{u \in U\_\text{uncovered} : uv \in E \text{ and } |\deg(u) - \deg(v)| \le 1\}|.$$

Approximation algorithms provide a middle ground by offering provable performance bounds for related domination problems. Klasing and Laforest (2004) show that strengthened domination variants can be difficult but still amenable to approximation strategies. Parameterized algorithms provide another route. Instead of measuring complexity only by the number of vertices n, they analyse complexity in terms of a parameter k, such as the target solution size, treewidth or maximum degree. Cygan et al. (2015) explain how this logic can make hard graph problems tractable for small parameters.

$$T(n, k) = f(k) \cdot n^c, \text{ where k is the parameter and c is constant.}$$

For equitable domination in bipartite and multilayer networks, the most useful algorithmic strategy is often hybrid. Exact models should be used for benchmark cases; greedy heuristics should be used for large-scale exploratory solutions; approximation should be used when guarantees are required; and parameterized methods should be used when the network has exploitable structure. This layered methodological strategy mirrors the layered nature of the networks themselves.

Figure 3. Illustrative algorithmic effort for exact, greedy and parameterized methods. Values are scaled for conceptual comparison, not empirical runtime data.

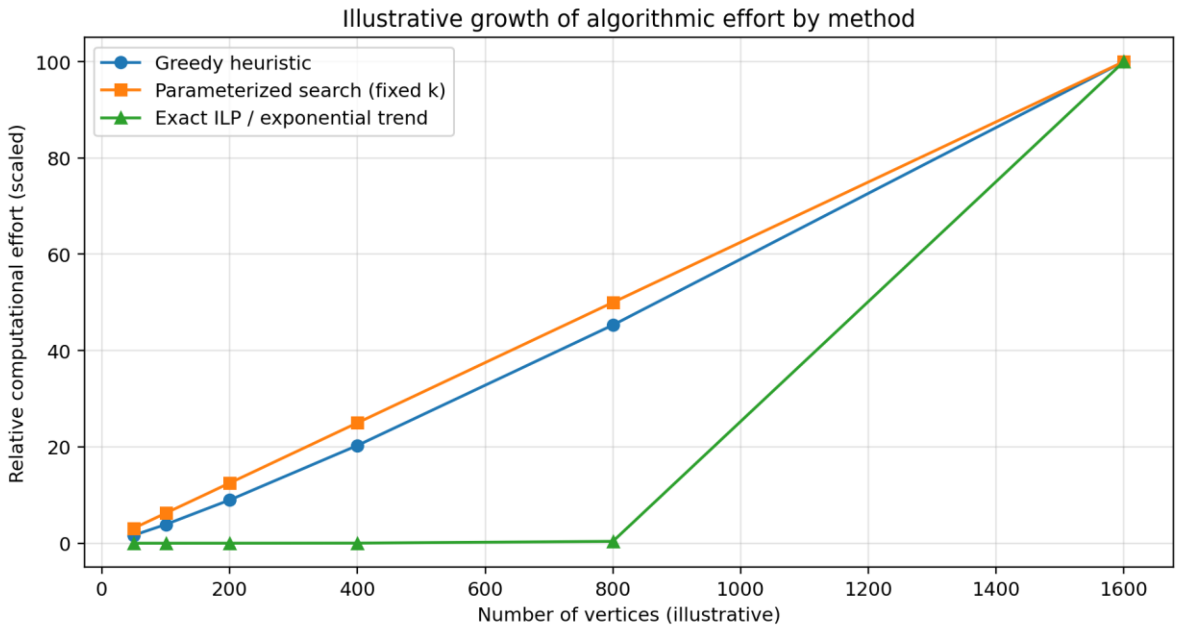


Table 3. Algorithmic approaches for equitable domination

Method	Core idea	Strength	Limitation	Suitable use
Exact ILP	Binary decision variables and full constraints	Optimal solution	Poor scalability	Small networks and benchmarks
Greedy heuristic	Select best local coverage score	Fast and interpretable	No universal optimality guarantee	Large exploratory networks
Approximation	Bounded-ratio solution	Theoretical guarantee	May be conservative	Policy-sensitive coverage
Parameterized	Exploit solution size or structure	Efficient for small parameters	Depends on structural assumptions	Sparse or bounded-width networks
Hybrid	Combine greedy seeding with exact refinement	Practical balance	Needs careful validation	Applied multilayer systems

8. Complexity Analysis

The classical dominating set problem is well known as a difficult combinatorial optimization problem. Many domination variants remain NP-hard, even when restricted to structured graph classes. The complexity literature on k-tuple domination, semipaired domination, secure domination and weakly closed graphs demonstrates that strengthened coverage requirements usually increase the difficulty of computation (Klasing & Laforest, 2004; Henning et al., 2019; Kumar & Reddy, 2020; Lokshtanov et al., 2021). Equitable domination is expected to share this difficulty because it contains a coverage requirement plus a compatibility condition.

Bipartite structure does not automatically make domination easy. Many domination-related problems remain hard in bipartite graphs or in subclasses of bipartite graphs. The reason is that bipartite graphs can still encode complex covering and selection problems. However, bipartite structure can help design better algorithms when the network has additional properties such as bounded degree, convexity, interval structure or small treewidth.

Multi-layered networks add another source of complexity. If each physical vertex appears in multiple layers, the number of vertex-layer states may grow as $|V| \times |L|$. Interlayer edges can also increase the size of the neighbourhood that must be checked. Aggregating layers may reduce computational cost, but it can also destroy important information. The core complexity trade-off is therefore between representational accuracy and algorithmic feasibility.

$|V\_M| = |V| \times |L|$, where $|L|$ is the number of layers.

Complexity pressure ≈ coverage constraints + equity constraints + layer-coupling constraints.

A practical implication follows: researchers should not only report the value of $\gamma_e(G)$ or $\gamma_e(M)$, but also report the modelling assumptions used to generate it. In a multilayer environment, the domination number can vary substantially depending on whether coverage is allowed within layers only, across all layers, or only through specified interlayer coupling edges.

9. Applications

In sensor networks, equitable domination can support balanced sensor placement. A selected sensor should not merely cover targets; it should have a comparable communication or connectivity level with the targets it monitors. This can reduce overload on a few high-degree sensors and improve robustness.

In social networks, domination models can identify influential actors who cover a larger community. Equitable domination adds caution by avoiding influence models that rely exclusively on elite hubs. It may highlight actors whose degree is compatible with the neighbourhood they influence, which can be useful for community-level communication campaigns.

In transportation networks, stations or routes can be selected to cover surrounding nodes. A multilayer model may combine bus, metro, road and pedestrian layers. Equitable domination can prevent a major hub in one layer from being treated as a universal service point if it does not provide compatible access in another layer.

In cybersecurity, nodes selected for monitoring can dominate neighbouring devices or service modules. Equitable domination can be interpreted as compatibility between monitoring capacity and monitored exposure. Multilayer models are especially relevant because cyber systems often contain physical, logical and application layers.

In biological or ecological networks, domination can support the identification of control species, genes or modules. Multi-layered ecological networks can represent different interaction types, such as mutualism, predation or competition. Layer-aware domination may provide more careful intervention targets than single-layer aggregation.

10. Discussion

The main conceptual contribution of equitable domination is that it reframes network coverage as compatible coverage. Ordinary domination asks whether a selected vertex can reach another vertex. Equitable domination asks whether the selected vertex can reach it under a local structural balance condition. This is particularly important where network control, service delivery or monitoring should not become excessively hub-centred.

In bipartite networks, this balance has a role-based meaning. Vertices on one side may represent providers and vertices on the other side may represent recipients. A small dominating set may be efficient but unfair or fragile if it assigns too much service responsibility to structurally mismatched nodes. Equitable domination therefore acts as a simple mathematical proxy for balanced assignment.

In multilayer networks, the value of equitable domination is even broader. It creates a framework for selecting nodes that are effective within and across layers while respecting local degree compatibility. The multilayer perspective helps prevent errors caused by aggregation. A node that appears central in an aggregated graph may not dominate effectively in the specific layer where service or control is required.

The limitation is that equitable domination also increases modelling and computational complexity. Degree compatibility may force larger dominating sets, and layer constraints may multiply the number of variables. This means that equitable domination should not be used mechanically. It is most useful when fairness, robustness or compatibility are central to the application.

11. Findings

- Equitable domination strengthens ordinary domination by adding a degree-compatibility condition.
- Bipartite networks provide a natural setting for equitable domination because they represent role-separated coverage systems.
- Multi-layered networks require layer-aware domination models because aggregation can distort coverage relationships.
- Exact optimization is useful for small networks but becomes difficult at scale.
- Greedy and hybrid algorithms are practically important for large applied networks.

• Parameterized approaches offer a promising direction when network structure or solution size is small.
• Equitable domination is relevant in sensor placement, public service coverage, social influence, cybersecurity and transport planning.
• Future research should develop benchmark datasets and test equitable domination on real bipartite and multilayer networks.

12. Recommendations

• Researchers should report both the ordinary domination number and the equitable domination number to show the cost of fairness-sensitive coverage.
• Algorithmic studies should compare exact, greedy and parameterized solutions on the same network instances.
• Bipartite applications should specify whether selection is allowed from one partition or from both partitions.
• Multilayer studies should clearly define whether domination is intra-layer, interlayer or fully layer-coupled.
• Applied studies should validate whether degree compatibility is a meaningful proxy for capacity, influence or service equivalence in the specific domain.
• Future mathematical work should examine approximation bounds and fixed-parameter tractability for equitable domination in structured bipartite graph classes.

13. Conclusion

Optimization and equitable domination provide an important bridge between graph theory, algorithm design and practical network applications. In bipartite networks, they model fair coverage between two different types of entities. In multi-layered networks, they extend domination from simple adjacency to layer-aware structural control. The degree-equitable condition gives domination a fairness-oriented interpretation by ensuring that selected and covered vertices remain locally compatible. This makes the concept relevant for service allocation, sensor placement, social influence, transport systems and cybersecurity monitoring.

At the same time, equitable domination is computationally demanding. Exact solutions may be infeasible for large networks, and multilayer structures increase both variables and constraints. Therefore, the most realistic research direction is not to rely on a single algorithm but to use a combination of exact models, heuristics, approximation analysis and parameterized techniques. The paper concludes that equitable domination should be treated as a flexible modelling framework: mathematically precise, computationally challenging and practically valuable when fair and robust coverage is required.

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